Exercise 74

Where is the function $h(x) = |x-1| + |x+2|$ differentiable? Give a formula for h' and sketch the graphs of h and h'.

Solution

Rewrite the function for $h(x)$.

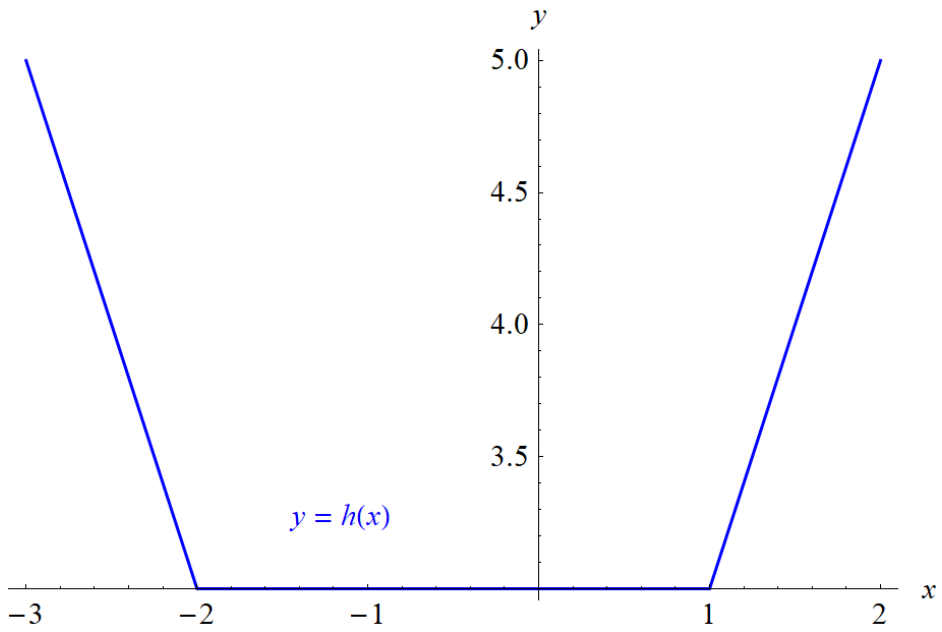
$$h(x) = |x-1| + |x+2| = \begin{cases} x-1 & \text{if } x-1 \geq 0 \\ -(x-1) & \text{if } x-1 < 0 \end{cases} + \begin{cases} x+2 & \text{if } x+2 \geq 0 \\ -(x+2) & \text{if } x+2 < 0 \end{cases}$$

$$= \begin{cases} x-1 & \text{if } x \geq 1 \\ 1-x & \text{if } x < 1 \end{cases} + \begin{cases} x+2 & \text{if } x \geq -2 \\ -x-2 & \text{if } x < -2 \end{cases}$$

$$= \begin{cases} (1-x)+(-x-2) & \text{if } x < -2 \\ (1-x)+(x+2) & \text{if } -2 \leq x \leq 1 \\ (x-1)+(x+2) & \text{if } x > 1 \end{cases}$$

$$= \begin{cases} -2x-1 & \text{if } x < -2 \\ 3 & \text{if } -2 \leq x \leq 1 \\ 2x+1 & \text{if } x > 1 \end{cases}$$

Below is a graph of $h(x)$ versus x.



Although the function is continuous, there are kinks in the curve at $x = -2$ and $x = 1$, which means the slope (or derivative) is undefined there. That is, h is not differentiable at 1 and 2.

The derivative of h is

$$h'(x) = \begin{cases} -2 & \text{if } x < -2 \\ 0 & \text{if } -2 < x < 1 \\ 2 & \text{if } x > 1 \end{cases},$$

and its graph versus x is shown below.